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A mathematical study of blood flow through radially non-symmetric multiple stenosed arteries under the influence of magnetic field

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Abstract

In the present study a mathematical model is proposed to describe blood flow through an axially non-symmetric but radially symmetric stenosed artery when blood is represented as power-law fluid and a uniform magnetic field is applied on the flow. The effect of magnetic field is considered in the transverse direction of blood flow and viscosity of blood is taken as radial co-ordinate dependent. The expression for velocity, resistance to flow and wall shear stress is derived. It is observed that the magnitudes of the blood flow characteristics significantly increase within the red cell concentration, which is depending on hematocrit value of blood. The importance of the decreasing velocity and resistance with increasing Hartmann numbers and stenosis shape parameter is also pointed out. Resistance to flow and velocity reduces the abnormalities of the artery in the presence of magnetic field.

Keywords: Magnetic field, Resistance to flow, Shape of stenosis, Wall shear stress, Viscosity of blood, Power-law fluid.

1. Introduction

Among all the fatal diseases of the human body, circulatory disorders are a still a major cause of death. A systematic study on the rheological and hemodynamic properties of blood and blood-flow could play a significant role in the basic understanding, diagnosis and treatment of many cardiovascular, cerebro-vascular and arterial diseases. It is well known that stenosis (Fig. 1.) (narrowing in the local lumen in the artery) is responsible for many cardiovascular diseases. When the degree of narrowing becomes significant enough to impede the flow of blood from the left ventricle to the arteries, heart problems develop. While the exact mechanism of the formation of stenosis in a conclusive manner remains somewhat unclear from the standpoint of physiology and pathology. The abnormal deposition of various substances like cholesterol, fat on the endothelium of the arterial wall, and proliferation of connective tissues accelerate the growth of the disease. Plaques are thereby formed and lead to serious circulatory disorders. Plaque forms when cholesterol, fat and other substances build up in the inner lining of the artery. This process is called carotid circulatory disorders. It greatly disturbs the normal blood flow leading to malfunction of the hemodynamic system and cardio vascular system.

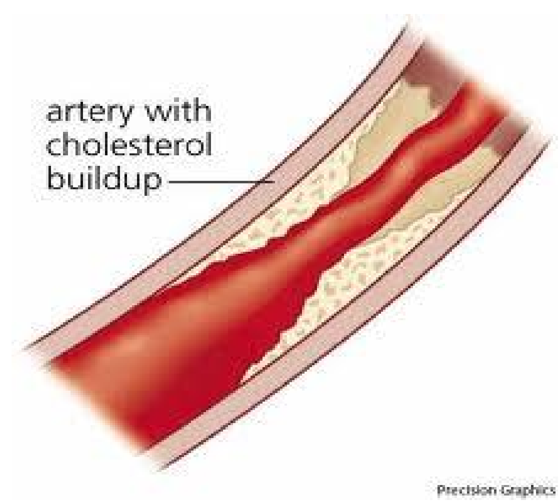

Fig.1. Atherosclerotic artery

One may expect that if such an event occurs, the flow characteristics in the vicinity of the resulting protuberance may be significantly altered. In an arterial constriction blood viscosity increases due to the conservation of mass, producing increased wall shear stress in the region of the blood acceleration. Several attempts have been made in the literature to study the effect of stenosis on the blood flow characteristics, including the important contribution of Young [2], Nerem [3], Haldar [4], Bitoun and Bellet [5], Chakravarty [6], Srivastava and Saxena [7], Srivastava [8], Srivastava [9]. Shukla et al., [10] have studied the effect of stenosis on the resistance to flow through artery by considering the behaviour of blood as a power-law fluid model. A little attention [10, 11, 12] have been made to study the effect of magnetic field on physiological fluid flows. It has been found that with the help of magnets, the flow of blood in arteries is properly regulated with the regulation in flow [13]. It has also been repoted by Barnothy [14] that the biological systems are affected by magnetic field. Sud and Sekhon [15] and Tzitzilakis [16] have analysed the effect of magnetic on blood flow through the human arterial system. Amos and Ogulu [17] concluded a similar study in which they obtained numerical results for the stream function and vorticty using the Gelarkin technique of the finite element method. The effect of an externally applied magnetic field over the flow characteristics of blood in a single stenosed artery has been analysed by Haldar [18].

2. Formulation of the problem

Consider the axisymmetric flow of blood in a uniform circular tube with an axially non-symmetric but radially symmetric mild stenosis. The geometry of the wall surface can be described as (Fig. 2):

$$\frac{R'(z)}{R_0}=\begin{pmatrix} 1-A[L_0'^{(m-1)}(\alpha z-k'd'-(k'-1)L_0') \\ -(\alpha z'-k'd'-(k'-1)L_0')^m] \end{pmatrix}; \qquad k(d'+L_0')-L_0' \le \alpha z' \le k'(d'+L_0') \qquad (1)$$

$$=1; \qquad \text{otherwise}$$

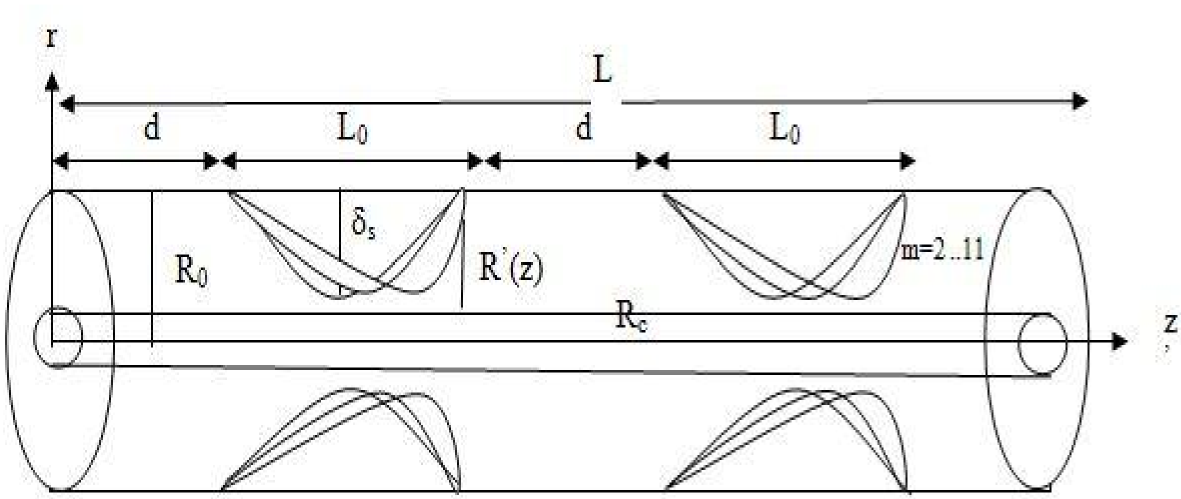

Fig.(2) Geometry of Stenosed artery

where $R'(z)$ is the radius of the artery with stenosis, R_0 is the constant radius of the artery. L_0 is the stenosis length and d indicates the stenosis location.

Here axially symmetric stenosis occurs when m = 2, the parameter A is as:

$$A = \frac{\delta_S}{R_0 (L_0^{'})^m} \frac{m^{m/(m-1)}}{(m-1)},$$

where $_s$ denotes the maximum height of stenosis at $z^{'} = \left[\frac{kd^{'} + (k-1)L_0^{'} + L_0^{'} / m^{1/(m-1)}}{\alpha} \right]$.

Power-law fluid model:

The stress-strain relation of a Power-law fluid is given by,

$$\left(-\frac{du^{'}}{dr^{'}} \right) = \left(\frac{2^{'}}{\mu^{'}} \right)^{1/n^{'}}, \tag{2}$$

Where u is the axial velocity, μ is the consistency function and n is the power-law exponent.

Following boundary conditions are introduced to solve the above equations,

$$\begin{aligned}
&\left(\frac{\partial u^{'}}{\partial r^{'}} \right) = 0 \quad \text{at} \quad r^{'} = 0, \quad u^{'} = 0 \quad \text{at} \quad r^{'} = R(z) \\
&\tau^{'} \ \text{is finite} \quad \text{at} \quad r^{'} = 0 \\
&P^{'} = P_0 \quad \text{at} \quad z^{'} = 0, \quad P^{'} = P_L \quad \text{at} \quad z^{'} = L
\end{aligned} \tag{3}$$

Governing equations:

Governing equation can be written as:

$$\left(-\frac{\partial P^{'}}{\partial Z^{'}} \right) + \frac{1}{r^{'}} \frac{\partial}{\partial r^{'}} \left(\mu^{'} r^{'} \frac{\partial u^{'}}{\partial r^{'}} \right) + \left(J^{'} \times B^{'} \right) = 0 \tag{4}$$

$$J^{'} = \sigma (E^{'} + u^{'} \times B^{'}) \ \text{and} \ \mu^{'} = \mu_0 \left(\frac{r^{'}}{R_0} \right)^{(-M)}$$

Where $E^{'}$ is electric field, $B^{'}$ is magnetic field, σ is electric conductivity, $J^{'}$ is magnetic flux and M is a parameter depending upon the hematocrit value.

Boundary conditions:

Following boundary conditions are introduced to solve the above equations:

$$\begin{aligned}
&\left(\frac{\partial u}{\partial r} \right) = 0 \quad \text{at} \quad r = 0 \\
&u = 0 \quad \text{at} \quad r = R(z) \\
&\tau \ \text{is finite} \quad \text{at} \quad r = 0 \\
&P = P_0 \quad \text{at} \quad z = 0 \\
&P = P_L \quad \text{at} \quad z = L
\end{aligned} \tag{5}$$

Non dimensional scheme:

$$R=\left(\frac{R'}{R_0}\right),\mu=\left(\frac{\mu'}{\mu_0}\right),r=\left(\frac{r'}{R_0}\right),L_0=\left(\frac{L_0'}{L}\right),\sigma=\left(\frac{\sigma'}{R_0}\right),$$
$$Re=\left(\frac{\rho U_0 R_0}{\mu_0}\right),d=\left(\frac{d'}{L}\right),z=\left(\frac{z'}{L}\right),P=\left(\frac{P'}{\rho U_0^2}\right),u=\left(\frac{u'}{U_0}\right) \tag{6}$$

The governing equations and boundary conditions are transformed to:

$$\frac{R(z)}{R_0}=\begin{pmatrix}1\text{-}A[L_0^{(m-1)}(\alpha z\text{-}kd\text{-}(k\text{-}1)L_0) \\ \text{-}(\alpha z\text{-}kd\text{-}(k\text{-}1)L_0)^m]\end{pmatrix}; \qquad k(d+L_0)\text{-}L_0\le\alpha z\le k(d+L_0) \tag{7}$$
$$=1; \qquad \text{otherwise}$$

Where, $A=\dfrac{\delta}{R_0L_0^m}\dfrac{m^{m/(m-1)}}{(m-1)}$

$$r^{-M}\left(\frac{\partial^2 u}{\partial r^2}\right)+(1-M)r^{-(1+M)}\left(\frac{\partial u}{\partial r}\right)-H^2u=Re\,£\left(\frac{\partial p}{\partial z}\right) \tag{8}$$

Where, $J=\sigma(E+u\times B)$, $\mu=r^{-M}$ and $H^2=\left(\dfrac{B_0^2R_0^2\sigma}{\mu_0}\right)$

$$\left(-\frac{du}{dr}\right)=\frac{Re\,\varepsilon}{H^2}\left(\frac{2}{\mu}\right)^{1/n}, \tag{9}$$

$$\left(\frac{\partial u}{\partial r}\right)=0 \qquad \text{at} \qquad r=0$$
$$u=0 \qquad \text{at} \qquad r=R(z) \tag{10}$$

3. Solution of the problem:

Solving for u from equation (8) and (9) and using boundary conditions (10), obtains,

$$u=\left(\frac{R_e\varepsilon}{2\mu}\right)^{1/n}\left[1+\frac{H^2(r-R_c)^2}{(1+R^2)}+\frac{\left(8(r-R_c)+H^2\right)H^2}{(1+R^2)^2}\right]^{2/n}\left(\frac{\partial P}{\partial Z}\right)^{(1/n)}$$
$$+\frac{\left(R_e\varepsilon\dfrac{\partial P}{\partial Z}\right)^{(1/n)}}{(4\mu)^{(1/n)}\left(1+R_c^{\,2}\right)^{(1/n)}}\left[r^2+\frac{\left(8R_c+H^2\right)}{\left(1+R_c^{\,2}\right)}r^4+\frac{\left(15(r-R_c)+H^2\right)\left(8(r-R_c)+H^2\right)}{\left(1+R_c^{\,2}\right)^2}\right]^{(1/n)} \tag{11}$$

The flow rate for the blood flow with transverse magnetic field is,

$$Q=\int_0^R 2\,\pi\,u\,r\,dr \tag{12}$$

By the help of equation (11) and equation (12), flow rate can,

$$Q=\left(\frac{R_e\varepsilon}{2\mu}\left(\frac{\partial P}{\partial Z}\right)\right)^{1/n}\left[\begin{array}{l}\frac{(4\mu R)^{(1/n)}}{R^2}+\frac{(8R+H^2)}{(1+R)}r^4+\frac{(15R+H^2)(8R+H^2)}{R^2}+\\ \frac{H^2R^2}{(1+R^2)}+\frac{(8R+H^2)H^2}{(1+R^2)^2}\end{array}\right]^{[(1/n)+1]} \quad (13)$$

From equation (13) pressure gradient is written as follows,

$$\left(\frac{\partial P}{\partial Z}\right)=\left(\frac{R_e\varepsilon}{2\mu}Q\right)^{n}\left[\begin{array}{l}\frac{(n\mu R)^{n}}{R^2}+\frac{(HR_e{}^2+1)^n}{(2+R)^2}+\frac{(4n+H^2)(H^2+2n)}{R^2}+\\ \frac{nH^2R_eR^2}{(1+R^2)}+\frac{(2\mu R+H^2)}{(1+R^2)^2}\end{array}\right]^{[(n+1]} \quad (14)$$

The dimensionless expression for resistance to flow, using

$$\lambda=\left(\frac{P_i-P_0}{Q}\right), \quad (15)$$

Using the boundary conditions, P_i is pressure at z = 0 and P_0 is the pressure at z = L. The resistance to flow can be written as:

$$\begin{aligned}\lambda=-\left(\frac{R_e\varepsilon}{n\pi}\right)^{n}&\left\{\frac{L}{R_0}\left[1-\frac{3\delta}{2R_0}+\frac{9\delta}{8R_0^2}+A_1\left(\frac{5\delta}{2R_0}-1\right)\right]^{n}\right.\\ &+\frac{L_0}{2\pi R_0}\left[\frac{3\delta_S}{2R_0}-\frac{6\delta^2}{4R_0^2}+A_1\left(\frac{20\delta^2}{4R_0^2}-\frac{5\delta}{2R_0}\right)\right]^{n}\left((L-d)^{(m+1)}-\frac{L_0}{2}\right)^{n}\\ &\left.+\frac{L_0}{4\pi R_0}\frac{7\delta^2}{8R_0^2}\left(\frac{L_0}{2}\right)+\frac{L_0}{2\pi R_0}\left[\frac{3\delta}{2R_0}+\frac{6\delta^2}{4R_0^2}+A_1\left(\frac{20\delta^2}{4R_0^2}-\frac{5\delta^2}{2R_0}\right)\right]^{n}\right. \end{aligned} \quad (16)$$

$$\text{where } A_1=R_e\varepsilon\left[R^2+\frac{(8+H^2)R^4}{(1+R^2)}+\frac{(8+H^2)(15+H^2)R^6}{(1+R^2)^2}\right]/\left[\frac{H^2R^2}{(1+R^2)}+\frac{(8+H^2)H^2R^4}{(1+R^2)^2}\right]$$

The shearing stress at the wall can be written as:

$$\tau=-\mu^{3n}\left\{\begin{array}{l}\left[\frac{2R_eH^2}{R^2}+\frac{4R^3(8+H^2)}{(1+R^2)^2}\right]^{3n}\left(\frac{\partial P}{\partial Z}\right)^{3n}\\ +\left(\frac{R_e\varepsilon\frac{\partial P}{\partial Z}}{(1+R^2)}\right)^{3n}\end{array}\left[\frac{4R^3(8+H^2)}{(1+R^2)}+\frac{(8+H^2)(15+H^2)}{(1+R^2)^2}\right]^{3n}\right\} \quad (17)$$

$$\mu_{app}=1/\left(R(z)/R_0\right)^4 f(\bar{y}) \quad (18)$$

$$\text{where } f(\bar{y})=R_e\varepsilon\left[\frac{(15+H^2)R^2}{(1+R^2)}+\frac{(8+H^2)(15+H^2)R^6}{(1+R^2)^2}\right]/\left[\frac{(8+H^2)H^2R^4}{(1+R^2)^2}\right]$$

4. Results and Discussions:

In order to have estimate of the quantitative effects of various parameters such as Hartmann number (H= 0, 0.2, .06, 1), red cell (M= 0, 2, 4), stenosis shape parameter (m= 2...11), flow behavior index (n= 1, 2/3, 1/3), α=1.0,1.1,1.2, and k=1,2 involved in the analysis computer codes were developed and to evaluate the analytical results obtained for resistance to blood flow, velocity profile and wall shear stress for diseased system associated with stenosis due to the local deposition of lipids have been determine. The results are shown in Fig3-6 by using the values of parameter based on experimental data in stenosed artery.

Fig.3 reveals the variation of resistance to flow (λ) with stenosis shape parameter (m). It is observed that the resistance to flow (λ) decreases as stenosis shape parameter (m) increases, maximum resistance to flow (λ) occurs at (m = 2), i. e. in case of symmetric stenosis. The result is consisting with the result of [111]. Fig.4 consists the variation of resistance to flow (λ) with axial distance for different values of Hartmann numbers (H) and stenosis size (δ/R_0). It is evident that resistance to flow increases as axial distance increases [15]. Resistance to flow increase as stenosis grows or radius of artery decreases (this referred to as Fahraeus- Lindquist effect in very thin tubes).

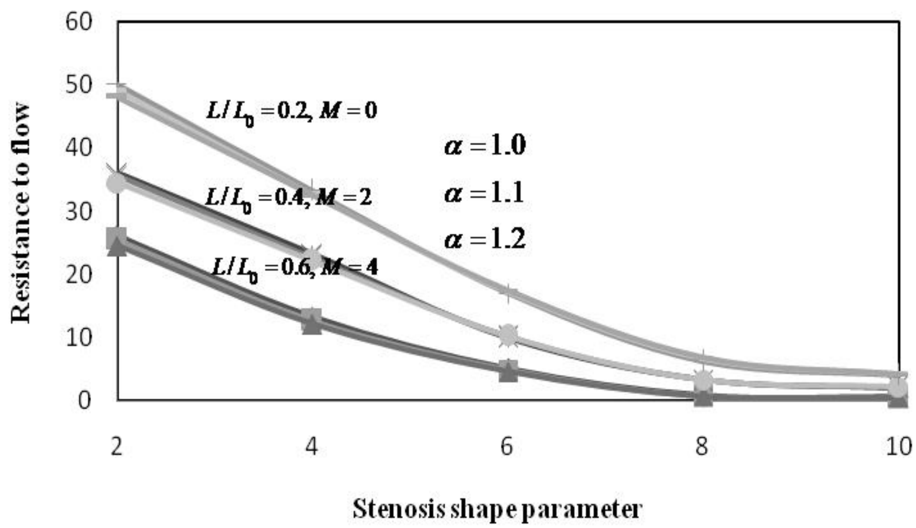

Fig. 3. Variation of resistance to flow with stenosis shape parameter for different values of stenosis length and α

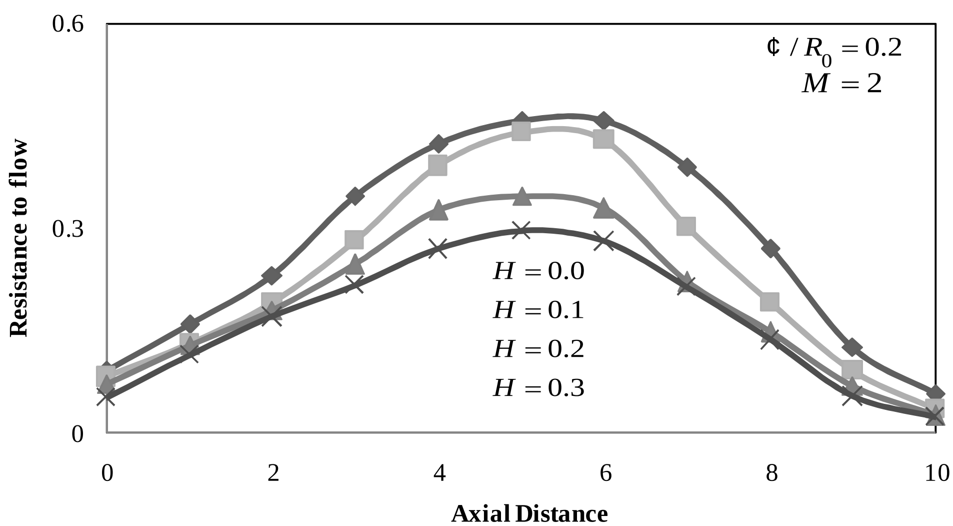

Fig. 4. Variation of resistance to flow with axial distance (z) for different values of Hartmann number

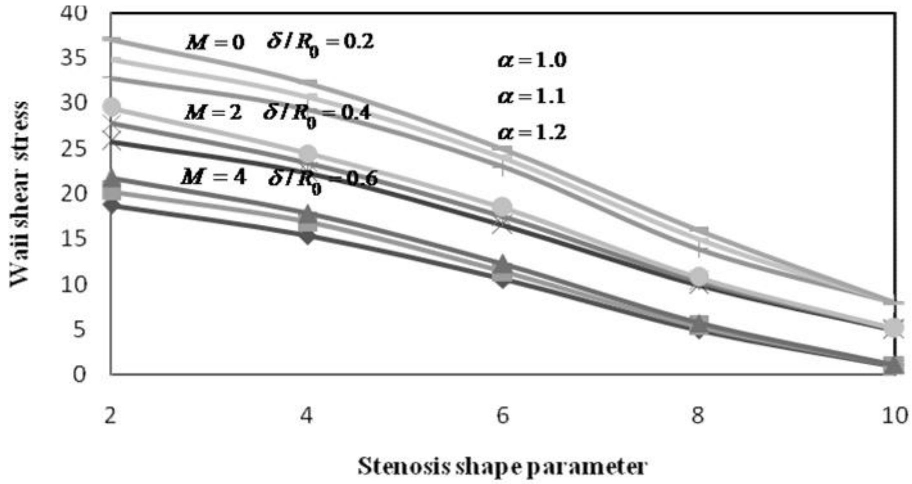

Fig. 5. Variation of wall shear stress with stenosis shape parameter for different values of stenosis size and α

Fig.5 describes the variation of wall shear stress (τ) with stenosis shape parameter and stenosis size. This figure depicts that wall shear stress (τ) increases as stenosis size increases but decreases as stenosis shape parameter increases. These results are consistent to the observation of [12]. In Fig.6 the variation of apparent viscosity with stenosis shape parameter and stenosis size has been shown. This figure depicts that apparent viscosity increases as stenosis size increases but decreases as stenosis shape parameter increases. As the stenosis grows, the apparent viscosity increases in the stenotic region. These results are similar with the results of [14].

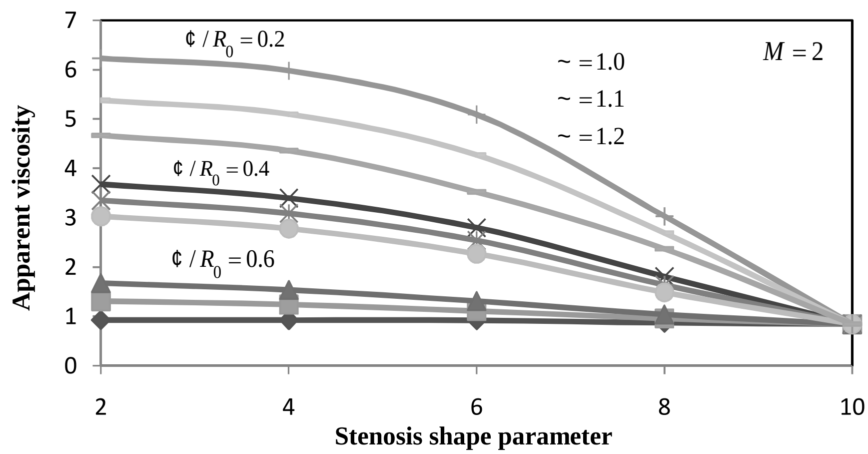

Fig .6. Variation of apparent viscosity with stenosis shape parameter

5. Concluding Remarks:

Blood flow through an artery mainly depends on the pressure gradient and resistance to flow. Resistance to flow increases as the stenosis grows and remains constant outside the stenotic region. If resistance to flow increases, it is more difficult for the blood to pass through an artery, result the flow decreases and heart has to work harder to maintain adequate circulation. In this present study, velocity profile, resistance to flow and wall shear stress are obtained when blood is assuming as power-law fluid (electrically conducting fluid) through an axially non-symmetric but radially symmetric stenosed artery, so that the effect of magnetic field on blood flow through an artery can be observed. Looking at the importance of the hydrodynamic factors in the understanding of blood flow and atherosclerotic diseases, it may be said that the present model could be useful for investigating blood flow through stenosed artery, in particular in diseased stage when blood is no longer a Newtonian fluid, it has yield stress with magnetic effects. It is noticed that the resistance to blood flow decreases for different magnetic fields and also decreases as stenosis shape parameter increases. The magnetic field is to decrease the resistance to flow due to irregular boundaries. So the magnetic field can be effectively utilized to deaccelerate the blood flow in flow problems.

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